МНК и метод Гаусса-Ньютона

Система уравнений и метод Ньютона

$$\begin{cases}\dfrac{\partial F(\vec{\beta})}{\partial \beta_1} = 0 \\ \dots \\ \dfrac{\partial F(\vec{\beta})}{\partial \beta_m} = 0\end{cases} \Rightarrow \frac{\partial F(\vec{\beta})}{\partial \beta_i} \approx \frac{\partial F(\vec{\beta}^{(0)})}{\partial \beta_i} + \sum_j \frac{\partial F(\vec{\beta}^{(0)})}{\partial \beta_i \partial \beta_j}\left(\beta_j - \beta_j^{(0)}\right) = 0$$

Матричная запись и метод Гаусса-Ньютона

$$\nabla F(\vec{\beta}^{(0)}) + H(\vec{\beta}^{(0)})\vec{p} = 0 \Rightarrow J^\top(\vec{\beta}^{(0)}) f(\vec{\beta}^{(0)}) + J^\top(\vec{\beta}^{(0)}) J(\vec{\beta}^{(0)})\vec{p} = 0$$

Градиент, якобиан и гессиан

$$(\nabla F)_i = \frac{\partial F}{\partial \beta_i} = \frac{\partial}{\partial \beta_i}\left(\sum_j f_j^2\right) = 2\sum_j f_j \frac{\partial f_j}{\partial \beta_i} = 2J^\top f; J_{ij} = \frac{\partial f_i}{\partial \beta_j} = \frac{\partial \varphi(\vec{\beta}, \vec{x_i})}{\partial \beta_j}$$

$$H_{ij} = \frac{\partial F(\beta)}{\partial \beta_i \partial \beta_j} = \frac{\partial}{\partial \beta_j}\left[2\sum_k f_k \frac{\partial f_k}{\partial \beta_i}\right] = 2\sum_k \left[\frac{\partial f_k}{\partial \beta_i}\frac{\partial f_k}{\partial \beta_j} + f_k \frac{\partial f_k}{\partial \beta_i \partial \beta_j}\right] \approx 2\sum_k \frac{\partial f_k}{\partial \beta_i}\frac{\partial f_k}{\partial \beta_j} = 2J^\top J$$